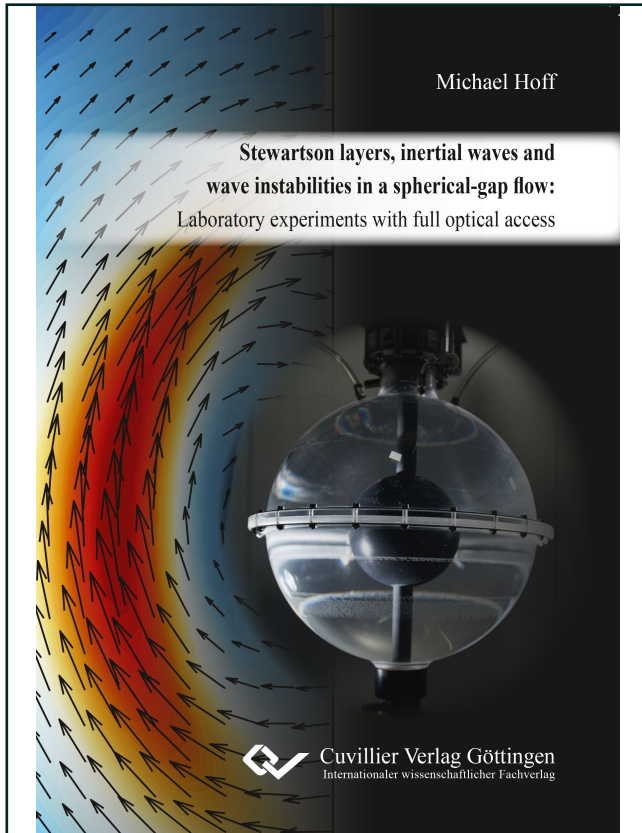




Michael Hoff (Autor)

# **Stewartson layers, inertial waves and wave instabilities in a spherical-gap flow**

Laboratory experiments with full optical access



<https://cuvillier.de/de/shop/publications/7640>

Copyright:

Cuvillier Verlag, Inhaberin Annette Jentsch-Cuvillier, Nonnenstieg 8, 37075 Göttingen, Germany

Telefon: +49 (0)551 54724-0, E-Mail: [info@cuvillier.de](mailto:info@cuvillier.de), Website: <https://cuvillier.de>



# Contents

<b>Abstract</b>	<b>iii</b>
<b>Zusammenfassung</b>	<b>v</b>
<b>Contents</b>	<b>vii</b>
<b>List of Figures</b>	<b>xi</b>
<b>List of Tables</b>	<b>xv</b>
<b>Abbreviations</b>	<b>xvii</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Why spherical-gap geometry? . . . . .	1
1.2 About this thesis . . . . .	5
<b>2 Fundamentals</b>	<b>11</b>
2.1 Equations of motion . . . . .	11
2.1.1 Pressure gradient acceleration . . . . .	12
2.1.2 Gravitational acceleration . . . . .	12
2.1.3 Friction acceleration . . . . .	12
2.1.4 Apparent accelerations . . . . .	13
2.1.5 The Navier-Stokes equation . . . . .	15
2.2 The continuity equation . . . . .	15
2.3 Plane inertial waves in unbounded fluids . . . . .	16
2.3.1 The dispersion relationship . . . . .	17
2.3.2 Phase- and group velocity . . . . .	20
2.3.3 Limiting case of a high-frequency wave . . . . .	21
2.3.4 Limiting case of a low-frequency wave . . . . .	21
2.4 Inertial waves in bounded fluids . . . . .	23
2.4.1 Reflection of inertial waves at inclined boundaries . . . . .	23
2.4.2 Wave rays, closed orbits and attractors . . . . .	25
2.5 Inertial waves in spheres and spherical gaps . . . . .	27
2.5.1 Inertial modes in a sphere . . . . .	27
2.5.2 Inertial waves in a spherical gap - shear layers . . . . .	34



2.6	Taylor-Proudman theorem . . . . .	36
2.7	Over-reflection and Critical layers . . . . .	37
<b>3</b>	<b>Experimental setup</b>	<b>39</b>
3.1	Description of the apparatus . . . . .	40
3.2	Control unit . . . . .	42
3.3	Libration efficiency . . . . .	43
3.4	Calibration of rotation rate and libration . . . . .	45
3.4.1	Setup and method . . . . .	45
3.4.2	Calibration of the outer and inner sphere . . . . .	47
3.4.3	Calibration of the libration parameters . . . . .	48
3.4.4	Calibration of the differential rotation . . . . .	50
<b>4</b>	<b>Measurement techniques</b>	<b>53</b>
4.1	Laser-light-sheet technique . . . . .	54
4.2	Qualitative measurements in the meridional plane . . . . .	56
4.3	Quantitative measurements in the horizontal plane (planar PIV) . . . . .	57
4.3.1	General overview and fundamentals . . . . .	58
4.3.2	Setup and experimental procedure . . . . .	61
4.3.3	The GoPro camera and its properties . . . . .	62
4.3.4	The MatPIV toolbox and its application to the data . . . . .	64
4.3.5	Error estimation . . . . .	66
<b>5</b>	<b>Post-processing methods</b>	<b>69</b>
5.1	Discrete Fourier transformation . . . . .	69
5.2	Harmonic analysis . . . . .	69
5.2.1	Least-square method . . . . .	71
5.2.2	General form of the matrix equation . . . . .	71
5.2.3	Example with one dominant frequency . . . . .	72
5.3	Bispectral analysis - Bicoherence . . . . .	74
5.4	EOF and CEOF analysis . . . . .	77
5.5	Estimation of the linear propagator . . . . .	80
5.6	Estimation of principle oscillation patterns (POPs) . . . . .	83
5.7	Estimation of singular vectors (SVs) . . . . .	83
5.8	Relationship between SVs, POPs and adjoint POPs . . . . .	84
<b>6</b>	<b>The Stewartson layer</b>	<b>87</b>
6.1	About Stewartson layers and their instabilities . . . . .	87
6.1.1	What is the Stewartson layer? . . . . .	87
6.1.2	Instabilities of the Stewartson layer . . . . .	92
6.2	The classical Stewartson layer with differentially rotating boundaries . . . . .	95
6.2.1	Data and data processing . . . . .	95
6.2.2	The structure of the Stewartson layer . . . . .	97
6.2.3	Route to Stewartson-layer instability for increasing $ Ro $ . . . . .	101
6.2.4	The scaling-law of the Stewartson-layer instability . . . . .	103
6.2.5	Patterns of Stewartson-layer instability for $Ro < 0$ and $Ro > 0$ . . . . .	105
6.2.6	Interactions between mean flow and spiral waves . . . . .	108
6.3	The ‘new’ jet-like Stewartson layer for inner-sphere libration . . . . .	110

6.3.1	Data and data processing . . . . .	110
6.3.2	The Structure of the Stewartson layer . . . . .	113
6.3.3	Route to Stewartson-layer instability for high $\hat{\epsilon}_{\text{lib}}$ and small $\hat{\omega}_{\text{lib}}$ . . . . .	116
6.3.4	Patterns of Stewartson-layer instability for librating inner sphere . . . . .	119
6.4	Discussion and comparison . . . . .	123
<b>7</b>	<b>Linear inertial waves driven by inner sphere libration</b>	<b>129</b>
7.1	Introduction . . . . .	130
7.2	The ray-tracing model . . . . .	131
7.3	Data and data processing . . . . .	134
7.4	Linear inertial waves in the meridional plane . . . . .	135
7.5	Linear inertial waves in the horizontal plane . . . . .	139
7.6	Triad interactions with Rossby waves . . . . .	141
7.7	Summary and conclusion . . . . .	144
<b>8</b>	<b>Inertial modes driven by differential rotation</b>	<b>145</b>
8.1	Introduction . . . . .	146
8.2	Experimental setup and data processing . . . . .	148
8.2.1	Experimental setup . . . . .	148
8.2.2	Data and data processing . . . . .	150
8.3	The velocity spectrograms . . . . .	151
8.4	Inertial wave modes' detection and comparison with the literature . . . . .	154
8.5	The excitation of inertial modes – the presence of critical layers? . . . . .	159
8.6	Transition to small-scale turbulence . . . . .	160
8.6.1	Kinetic energy distribution - mode amplification and zonal mean flow enhancement . . . . .	161
8.6.2	Frequency shift of the dominant wave modes – Doppler effect . . . . .	162
8.6.3	The scaling-law of $Ro_c$ . . . . .	165
8.6.4	Increased turbulence in the inertial wave range . . . . .	167
8.6.5	Subharmonic instability as an explanation for the transition to wave turbulence? . . . . .	168
8.6.6	Non-modal growth as an explanation for the transition to wave turbulence? . . . . .	169
8.7	Preliminary results of accompanying numerical simulations . . . . .	173
8.8	Discussion and conclusion . . . . .	178
<b>9</b>	<b>Concluding remarks</b>	<b>185</b>
<b>A</b>	<b>Ray-tracing - detecting the correct intersection point</b>	<b>193</b>
A.1	Whole spherical gap without the shaft . . . . .	193
A.2	Half spherical gap including the shaft . . . . .	195
<b>B</b>	<b>Correction of radial distortion</b>	<b>197</b>
<b>C</b>	<b>Inertial mode spectrograms in the meridional plane</b>	<b>201</b>
	<b>Bibliography</b>	<b>203</b>
	<b>Acknowledgements</b>	<b>215</b>