



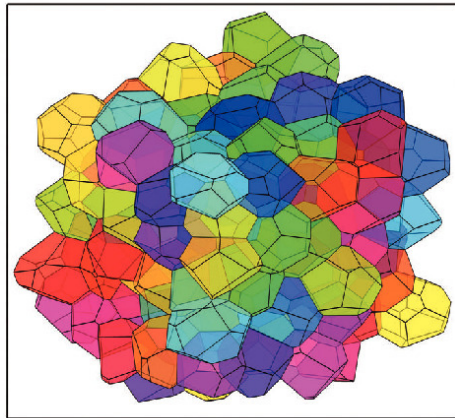
Luis Antonio Barraales Mora (Autor)  
**2D and 3D Grain Growth Modeling and Simulation**

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and Simulation

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Luis Antonio Barraales Mora



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