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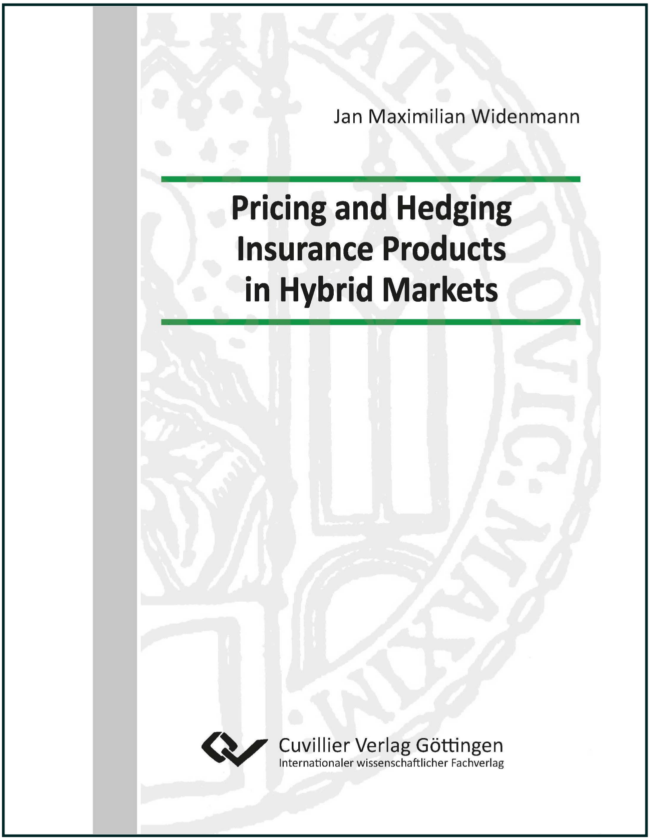
Pricing and Hedging Insurance Products in Hybrid Markets



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Introduction

Insurance in Hybrid Markets

The determination of risk-adjusted, fair insurance premiums and the management of the insurance company's risk exposure are core challenges in actuarial science. The developments on the insurance markets show the importance of well elaborated models which account for the economic environment of the insurance company. More explicitly, insurance companies

- can sell parts of their insurance risk by issuing insurance linked products on the financial markets, see Weber [95],
- can link the benefits of their insurance contracts to the performance of the assets on the stock markets by offering unit-linked insurance products, see Møller [71] or Vandaele and Vanmaele [93],
- have the possibility to invest in financial markets and hedge against their risks with financial instruments.

Insurance markets should therefore be considered as part of one big hybrid market in which appropriate pricing and risk-mitigation schemes are elaborated.

The present thesis introduces several frameworks in this context by applying no-arbitrage pricing schemes and quadratic hedging approaches to a large class of insurance contracts. All presented results are based on four research articles which have been submitted to refereed journals. The articles Biagini and Widenmann [14] as well as Biagini, Groll, and Widenmann [19] address the problem of flexibly modeling and pricing unemployment insurance contracts while the articles Biagini and Widenmann [15] as well as Biagini, Rheinländer, and Widenmann [20] cover the issues of optimally hedging insurance contracts in general settings.

A major novelty to the existing literature is to consider the underlying stochastic process, describing the insured person's progress in time of sojourning in the states, considered by an insurance policy, as an $\mathbb{F}$-doubly stochastic Markov chain. This class of stochastic processes was introduced in Jakubowski and Niewęgłowski [61] and extends the classic notion of Markov chains by including a reference filtration $\mathbb{F}$, characterizing e.g. additional market information. An important property of $\mathbb{F}$-doubly stochastic Markov chains is that they may admit matrix-valued stochastic intensity processes. This allows elaborating more flexible models compared to the results of e.g. Møller [72] where a (classical) Markov chain with a deterministic matrix-valued intensity function is considered. Well known examples of $\mathbb{F}$-doubly stochastic Markov chains are the reduced-form or hazard-rate models of credit risk or life insurance, provided they admit the so called immersion property. Here, the state space consists of only two states with the second state being absorbing such that there can only occur one transition in time. There exists a vast literature on pricing and hedging within this type of models, see e.g. Barbarin [6], Biagini and Cretarola [10, 11, 12], Biagini and Schreiber [13], Biagini et al. [18, 17], Bielecki and Rutkowski [24] or Bielecki et al. [21, 22], such that the consideration of (general) $\mathbb{F}$-doubly stochastic Markov chains covers and extends most of these works to a multi-state framework. The advantage that subsequent transitions can be considered facilitates the investigation of a larger class of insurance contracts, e.g. general payment protection insurance (PPI) contracts with the insured states “disabled”, “unemployed”, and “deceased”.

Pricing Unemployment Insurance Contracts

One of the investigated issues in this thesis is the modeling and calculation of fair insurance premiums for unemployment insurance contracts. More precisely, we consider a particular unemployment insurance product which pays a priori fixed, deterministic amounts to the insured person as soon as he gets unemployed and fulfills several other claim criteria. For example, one could think of PPI products against unemployment which are always linked to some payment obligation of the insured person. If an insured event occurs, the insurance company pays the (deterministic) installments during the respective period. Given the random claim payments of this insurance contract, we apply no-arbitrage pricing, in particular the benchmark approach with its real-world pricing formula, to determine risk-adjusted insurance premiums. The use of this approach for insurance applications is motivated as follows.

Pricing Insurance Contracts with the Benchmark Approach

Pricing of random claims has ever been one of the core subjects in both actuarial and financial mathematics and there exist various approaches for calculating (fair) prices. The actuarial way of pricing usually considers the classical premium calculation principles that consist of net premium and safety loading: if C describes a random claim which the insurance company has to pay (eventually) at time $T > 0$, then the premium $\pi(C)$ to be charged for the claim is defined by

$$\pi(C) = \underbrace{\mathbb{E}\left[\frac{C}{N_T}\right]}_{\text{net premium}} + \underbrace{A\left(\frac{C}{N_T}\right)}_{\text{safety loading}}, \tag{0.1}$$

where N is a discounting process, chosen according to actuarial judgement, see also Kull [67]. Note that the net premium is the expected value of $\frac{C}{N_T}$ with respect to the real-world (or objective) probability measure $\mathbb{P}$. Possible safety loadings are $A(\frac{C}{N_T}) = 0$ (net premium principle), $A(\frac{C}{N_T}) = a \cdot \mathbb{E}[\frac{C}{N_T}]$ (expected value principle, where $a \geq 0$), $A(\frac{C}{N_T}) = a \cdot \mathbb{V}ar(\frac{C}{N_T})$ (variance principle, where $a > 0$) or $A(\frac{C}{N_T}) = a \cdot \sqrt{\mathbb{V}ar(\frac{C}{N_T})}$ (standard deviation principle, where $a > 0$), see e.g. Rolski et al. [81]. The existence of a safety loading is justified by ruin arguments and the risk-averseness of the insurance company: the net premium principle with zero safety loading is unfavourable for the insurance company as the ruin probability of an increasing collective tends towards 50% (central limit theorem).

Widely used pricing approaches in finance base on no-arbitrage assumptions, see e.g. Black and Scholes [28] and Merton [70]. A financial market consisting of several primary assets is assumed to be in an economic equilibrium in which riskless gains out of nothing (arbitrage) by trading in the assets are impossible. A fundamental result in this context is then the essential equivalence of absence of arbitrage and the existence of an equivalent (local) martingale measure, i.e. a probability measure which is equivalent to the real-world measure $\mathbb{P}$ and according to which all assets, discounted with some numéraire process, are (local) martingales. There are different versions of this result which is often called the fundamental theorem of asset pricing (FTAP), see. e.g. Delbaen and Schachermayer [44], Delbaen and Schachermayer [45], Föllmer and Schied [53], Harrison and Pliska [57] or Kabanov and Kramkov [63].

Based on the FTAP, it can then be shown that at any time $t \in [0, T]$ an arbitrage-free price $\pi_t(C)$ of a (contingent) claim C (paid at time $T > 0$) is given by

$$\pi_t(C) := S_t^* \mathbb{E}_{\mathbb{Q}} \left[\frac{C}{S_T^*} \,\middle|\, \mathcal{G}_t \right] , \tag{0.2}$$

where $\mathbb{Q}$ is an equivalent (local) martingale measure, S^* the discounting process and $\mathbb{G} = (\mathcal{G}_t)_{t \in \mathbb{R}_+}$ the filtration which expresses the information on the market. Hence, the (new) discounted price process is assumed to follow a $(\mathbb{Q}, \mathbb{G})$-martingale.

Approaches which base on no-arbitrage assumptions are strong tools for the purpose of modeling price structures because they provide access to the powerful theory of (local) martingales. Other advantages are the dynamic description of price processes and the close connection to hedging.

From an economic point of view both the safety loading in Equation (0.1) and the change to an equivalent (local) martingale measure in Equation (0.2) express the risk-averseness of the insurance company. Moreover, there exist several works which connect actuarial premium principles with the financial no-arbitrage theory. The papers Delbaen and Haezendonck [43] and Sondermann [89] both describe a competitive and liquid reinsurance market in which insurance companies can "trade" their risks among each other. Since riskless profits shall be excluded also in this setting, the no-arbitrage theory applies and insurance premiums can be calculated by Equation (0.2). Both papers actually show that under some assumptions[1] there exist risk-neutral[2] equivalent (local) martingale measures which explain premiums of the form (0.1), so that these principles provide arbitrage-free prices, too. Further papers, connecting actuarial and financial valuation principles are e.g. Kull [67] and Schweizer [86]. Note that the possibility of trading insurance contracts rather applies to the secondary market in which insurance companies trade risks through reinsurance contracts or by securitization. In particular, Equation (0.2) provides reasonable prices for the secondary market. In a competitive primary market these prices generally constitute a good benchmark as well.

[1] The equivalent martingale measure is required here to be structure preserving, i.e. the claim process remains a compound Poisson process under $\mathbb{Q}$.

[2] For risk-neutral martingale measures, the numéraire process S^* is chosen to be the bank account S^0 in the domestic currency.

Martingale approaches are therefore very suitable for actuarial applications and get even more important for the evaluations in the aforementioned hybrid markets. Note that due to their unsystematic part of the risk, most insurance contracts are not replicable by other instruments on the hybrid market, which implies that the market is incomplete. As a consequence, there usually exist several equivalent (local) martingale measures, corresponding to the same numéraire, that guarantee the absence of arbitrage in the market. By Equation (0.2) it is then clear that defining a premium calculation principle in the market is equivalent to choosing a numéraire and an equivalent (local) martingale measure. The usual procedure in this context is to fix some numéraire and then to search for an appropriate measure. Examples, among others, are the minimal martingale measure and the minimal entropy measure. However, several measure choices seem not to be economically reasonable for hybrid markets. Moreover, it can be shown that for several insurance linked products with random jumps, the density of the minimal martingale measure may become negative and is therefore useless in the context of pricing.

To avoid these problems, we choose the benchmark approach for our pricing issue. This approach fixes the real-world probability measure $\mathbb{P}$ and tries to determine the numéraire process, more precisely a self-financing portfolio on the assets, called the $\mathbb{P}$-numéraire portfolio, such that the discounted (or benchmarked) primary assets become local martingales or, more generally, supermartingales. The existence and uniqueness of the $\mathbb{P}$-numéraire portfolio have been shown in sufficiently general settings, see Becherer [8] or Karatzas and Kardaras [65]. The existence of the $\mathbb{P}$-numéraire portfolio then guarantees the absence of arbitrage, which is defined in a stronger way than usual. There could still exist some weak form of arbitrage in the market, which would require negative portfolios of total wealth, however. In a realistic market model, such portfolios should be impossible due to the law of limited liability. A thorough description of the benchmark approach with its real-world pricing formula and its advantages for pricing insurance contracts are given in Sections 1.1.1 and 1.2.

Application to Unemployment Insurance

Choosing the benchmark approach for pricing unemployment insurance contracts intrinsically provides a first risk-factor for the insurance premium: the $\mathbb{P}$-numéraire portfolio. In the first pricing approach which bases on the results in Biagini and Widenmann [14], we assume the underlying $\mathbb{F}$-doubly stochastic Markov chain characterizing the employment-unemployment progress of an insured person in time to be time-homogeneous. A corresponding intensity is then still random but not varying over time. More precisely, we consider an $\mathbb{F}$-doubly stochastic Markov chain which is generated by a random matrix with entries, derived from the value of the $\mathbb{P}$-numéraire portfolio at maturity. In this setting several conditional independence and distribution properties can be used to transform the insurance premium into a conditional expectation with respect to the reference filtration $\mathbb{F}$ of some closed analytic expression. The insurance premium can then be further evaluated by specifying the reference filtration more precisely. In particular, we illustrate the evaluations when the reference filtration is generated by the $\mathbb{P}$-numéraire portfolio, considered to follow a Lévy process. Moreover, we show estimation and simulation results for the case when $\mathbb{F}$ is trivial, i.e. when the $\mathbb{F}$-doubly stochastic Markov chain is a (classical) time-homogeneous Markov chain.

This first model provides interesting and reasonable results and incorporates the $\mathbb{P}$-numéraire portfolio as a risk-process in an elegant way. In a second pricing approach which is based on the results in Biagini, Groll, and Widenmann [19] we generalize this framework in order to account for the aforementioned dependencies of the model in hybrid markets. We drop the assumption on time-homogeneity but assume the underlying $\mathbb{F}$-doubly stochastic Markov chain to be generated by intensity processes which are driven by individual-related as well as micro- and macro-economic covariate processes. In this framework it is generally not possible to obtain an analytic expression for the insurance premium similar to the first framework. Instead, the insurance premiums are derived by using Monte Carlo simulations.

In order to calibrate the price for the unemployment insurance products to real data, we estimate the intensity processes using Cox's proportional hazards model, see Andersen et al. [2] and Cox [37, 38]. The data set is provided by the "Institut für Arbeitsmarkt-und Berufsforschung" (IAB), the German institute for employment research, and contains a sample of integrated labor market biographies, including the duration of employment and unemployment periods between 1975-2008 of more than 1.5 million German individuals as well as several useful socio-demographic covariates, such as age, nationality, educational level, regional details, etc. In order to reflect additional dependencies of the intensity processes to macro-economic factors, we also incorporate further covariates such as time series for the MSCI-world returns and German unemployment rates.

An advantage of using Cox's proportional hazards model is the availability of adequate implementations, see for example the R-packages corresponding to Aalen et al. [1], de Wreede et al. [42] or Jackson [60]. Technically, the implemented estimators estimate the compensator processes of multivariate counting processes which count subsequent jumps of the same kind of some unspecified multi-state switching process. The estimators in this context are based on the martingale property of the compensated counting process. The question is, if one can define characteristics for the underlying multi-state switching process such that the corresponding compensator estimators also provide estimates for (parts of) these characteristics. A well known example in this context is an underlying multi-state switching process which follows a (classical) Markov chain with deterministic matrix-valued intensity function. Here, the intensity characterizes the compensator of the corresponding counting processes and vice versa such that the obtained estimators for the (deterministic) compensator provide estimators for the intensity function as well, see Andersen et al. [2]. Yet, to the best of our knowledge, a more general relation for stochastic compensators, particularly given by Cox's proportional hazards model, has not yet been established in the literature. Based on a martingale characterization in Jakubowski and Niewęgłowski [61], we bridge this gap and show that the class of $\mathbb{F}$-doubly stochastic Markov chains is the natural candidate to be considered as the underlying multi-state switching process. This relation can analogously be applied to general multiplicative hazards models as given in Andersen et al. [2].

In order to test the obtained estimation results, we apply conventional goodness-of-fit methods. The results generally show adequate performance of the estimated model parameters. We furthermore introduce a non-standard method for testing the applicability of the obtained parameters with respect to prediction by comparing actual and simulated jump times for selected paths of the data set. The results here show good predictive power which implies robustness of the Monte Carlo simulations to compute the premiums. A conclusive sensitivity analysis of the insurance premiums also confirms these findings.

In general, both frameworks represent flexible premium determination tools for unemployment insurance products since they incorporate risk factors. Moreover, they can be easily adapted to model and estimate stochastic intensities and dependence structures in many other different applications of financial and actuarial practice.

Quadratic Hedging of Insurance Contracts

The classic form of mitigating the risk exposure of an insurance company is to buy reinsurance such that parts of the risks are taken over by another insurance company. Another way is securitization. Here, parts of the risk are combined to a package of insurance linked securities which are then sold on the financial markets, see Weber [95]. The investors in these types of securities benefit from the low correlation between most types of insurance contracts to the classical types of securities like stocks or bonds. This way the insurance linked securities provide a good potential for diversification.

The third way of mitigating an insurance company's risk exposure is to hedge parts of the risk by appropriately trading in other assets. This particularly applies if the assets are correlated to the insurance contract's benefits or their (conditional) probability of occurrence. Practical examples in this direction are unit-linked life insurance products, where benefits depend on the performance of the assets, or the aforementioned unemployment insurance products, where the occurrence of the claim payment may depend to some extend on the performance of the stock markets. Moreover, there is an ongoing discussion about the introduction of so called longevity bonds which would establish the possibility for life insurance companies and pension funds to hedge parts of their longevity risk, see e.g. Biagini and Schreiber [13] or Blake et al. [31]. Longevity bonds typically involve a publicly accessible longevity index from which the mortality intensities for a wide range of age cohorts can be derived.

As already mentioned, due to their unsystematic risk part the insurance claims in consideration are not replicable by a self-financing trading strategy such that the hybrid market is incomplete. A reasonable method for optimally choosing an investment strategy is then important to cover at least parts of the risk. Well known and elaborated approaches in this context are based on quadratic optimality criteria. In the present thesis we apply mean-variance hedging and risk-minimization to a wide class of insurance contracts. For an overview on these quadratic hedging approaches we refer to Pham [75] or Schweizer [85].

To apply quadratic hedging for insurance contracts, we assume the discounted value processes of the primary assets on the hybrid market, i.e. the hedging instruments, to be nonnegative (local) martingales. This provides that the mean-variance and risk-minimizing hedging strategies are derived uniquely from the well known Galtchouk-Kunita-Watanabe (GKW-) decomposition, see Ansel and Stricker [4] or Kunita and Watanabe [68].

Mean-Variance Hedging for Life Insurance Products

In a first scenario which is based on the results in Biagini, Rheinländer, and Widenmann [20] we apply mean-variance hedging to both well known and newly introduced life insurance products by trading in longevity bonds. In particular, we consider pure endowments, i.e. contracts which pay out one unit if the insured person is alive at maturity, and term insurances, i.e. payments of one unit in case the insured person dies before the maturity of the contract. Moreover, we consider general life annuities, paying out continuous rates as long as the insured person is alive. In this context, we specify a new type of (insurance) contract which we call a gratification annuity. This insurance contract would pay increasing annuity rates, proportional to the conditional mortality probability of the insured person's own age cohort, inferred from the aforementioned longevity index. Broadly speaking, a policyholder gets gratified if the insured person is healthier or belongs to a sicker age cohort than was originally expected. The concept of a gratification annuity may also be interesting because it allows diversifying unsystematic insurance risk while transferring important parts of the systematic insurance risk to the policyholder, see also Norberg [74] and Wadsworth et al. [94] in this context. Therefore, such type of insurance contract could be interesting for the life insurance market.

The longevity bond as hedging instrument is modeled as an annuity, paying continuous rate payments proportional to the conditional survival probability, again inferable from a longevity index. There is an ongoing discussion in the literature, recommending the introduction of longevity bonds on capital markets, see e.g. Blake et al. [32]. Their appropriateness as hedging instrument for longevity risks has originally been proposed by Blake and Burrows [29].

The combined position in one of the life insurance contracts and the longevity bond also resembles various types of mortality swaps, see Dahl et al. [41] for a related concept, where the floating leg (realized mortality) is exchanged versus a fixed leg (related to some mortality projection). For a more detailed overview of the securitization of mortality risk we refer to Barrieu and Albertini [7], as well as Blake et al. [30].

Given that the underlying life history of an insured person follows an $\mathbb{F}$-doubly stochastic Markov chain with the two states "dead" and "alive", we implicitly work in the classical setting of reduced form or hazard-rate models, see Bielecki and Rutkowski [24]. We therefore use well known formulas which are specific for this two-state setting.

Under the assumption that the reference filtration $\mathbb{F}$ is generated by a one-dimensional Brownian motion W, the mean-variance hedging strategies are first calculated for a single life status and then generalized to hedging strategies for a whole portfolio of insured persons following the work of Biffis and Millossovich [26]. We remark that the GKW-decompositions obtained for the mortality claims could also be derived from the results in e.g. Barbarin [5] or Blanchet-Scalliet and Jeanblanc [33] for pure endowments, in Barbarin [5] for term insurance and in Barbarin [5] or Biagini and Cretarola [12] for general annuities. In our setting, however, we work under specific but still very general model assumptions which allow computing the GKW-decompositions explicitly. The setting furthermore allows illustrating the results for an affine specification of the mortality intensity process. This assumption is very popular in the literature about modeling mortality intensities and has been suggested for example in Biffis [25], Biffis and Millossovich [26], Dahl and Møller [40], Dahl et al. [41] or Schrager [83]. Here, we can relate the optimal hedging strategies to the solutions of well known Riccati ordinary differential equations (ODEs) and analyse the results with numerical simulations.

These simulations are carried out for two specifications of the mortality intensity, following in the first case an Ornstein-Uhlenbeck process and in the second case a Feller process. Both processes are considered to be non-mean-reverting, an assumption suggested by Luciano and Vigna [69] or Blake et al. [30]. In this context, we compare the optimal hedging strategies and their residual hedging error for a gratification annuity and a simple life annuity.

For further differences and advantages of the given framework to the ones existing in the vast literature on quadratic hedging of financial insurance derivatives, like e.g. in Barbarin [5], Dahl and Møller [40], Dahl et al. [41], Møller [71] or Møller [72], the interested reader is referred to Biagini, Rheinländer, and Widenmann [20].

Risk-Minimization for General Insurance Contracts

With similar techniques and ideas to the first hedging framework we generalize the setting and apply risk-minimization to a large class of insurance contracts, allowing also to model several consecutive state transitions of the insured person. The results here can similar be found in Biagini and Widenmann [15].

More specifically, we consider a general $\mathbb{F}$-doubly stochastic Markov chain which admits an intensity, and propose general insurance contracts as being defined by three types of insurance payments: state-dependent payments at maturity, state-dependent annuity-type payments, and (transition-dependent) payments at the transition-time from one state to another. This definition covers a large set of currently adopted insurance policies and is motivated by the definitions of rating sensitive claims in Jakubowski and Niewęgłowski [62] or defaultable claims in Bielecki et al. [23]. It covers the aforementioned insurance contracts of pure endowment, term insurance, general annuities and PPI as well as the concepts of insurance contracts, given in Møller [72] or Norberg [73].

Extending the results in Jakubowski and Niewęgłowski [62] who applied $\mathbb{F}$-doubly stochastic Markov chains in the context of replicating rating-sensitive financial claims we obtain the GKW-decomposition for the discounted value process of general insurance contracts with respect to a square-integrable $\mathbb{F}$-martingale.

In order to elaborate risk-minimizing hedging strategies it is then necessary to specify the underlying market. To this end, we assume that the reference filtration $\mathbb{F}$ is generated by an N-dimensional Brownian motion $\mathbf{W}$ and that the assets on the hybrid market are $\mathbb{F}$-adapted. In this setting the risk-minimizing hedging strategies are derived for general insurance contracts with a deterministic payment structure with respect to the assets on the market. Similar to the first framework on mean-variance hedging for life insurance contracts, the results are then further specified within a general affine setting for the intensity processes of the underlying $\mathbb{F}$-doubly stochastic Markov chain.

Guideline through the Thesis

Chapter 1

Chapter 1 introduces the basic notations, definitions and results which are used throughout the thesis. In Section 1.1, the notations and definitions for hybrid markets are given based on which the benchmark approach with its real-world pricing formula and the quadratic hedging approaches are overviewed in Subsections 1.1.1 and 1.1.2. Section 1.2 highlights the appropriateness of the benchmark approach and the quadratic hedging approaches for actuarial applications and connects their general concepts and results with each other.

Chapter 2

Chapter 2 is devoted to the pricing of unemployment insurance products. In Section 2.1 the specific form of the unemployment insurance contracts in consideration is presented based on which the corresponding insurance claim is specified. Using the real-world pricing formula of the benchmark approach, first evaluations of fair insurance premiums are made. In Section 2.2 a first framework for the insurance premiums within a time-homogeneous setting of the underlying $\mathbb{F}$-doubly stochastic Markov chain is presented. The specific results are then further illustrated within the Lévy process framework in Subsection 2.2.1 and within the classical Markov chain setting in Subsection 2.2.2. Section 2.3 provides the second framework for evaluating the insurance premiums. Here, Cox's proportional hazards model is connected with the class of $\mathbb{F}$-doubly stochastic Markov chains in Subsection 2.3.1. Subsection 2.3.2 then briefly overviews the estimators which are then applied to the dataset, described in Subsection 2.3.3. The estimation results are presented in Subsection 2.3.4 and tested on their appropriateness through several goodness-of-fit methods in Subsection 2.3.5. In Subsection 2.3.6, the estimates are used to evaluate the insurance premiums by Monte Carlo simulations.

Chapter 3

Chapter 3 covers the mitigation of longevity risk by trading in a longevity bond. Here, Section 3.1 establishes the specific modeling framework, used for deriving explicitly the mean-variance hedging strategies. These are established for a single life status in Section 3.2 and for insurance portfolios in Section 3.3. The results are then further illustrated