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# New numerical approaches to model hydraulic fracturing in tight reservoirs with consideration of hydro-mechanical coupling effects

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