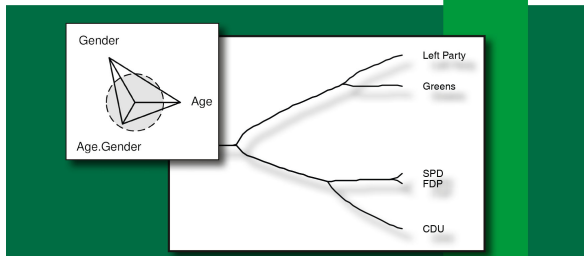




Gunther Schaubeger (Autor)  
**Regularization Methods for Item Response and  
Paired Comparison Models**

Gunther Josef Schaubeger

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Paired Comparison Models**



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