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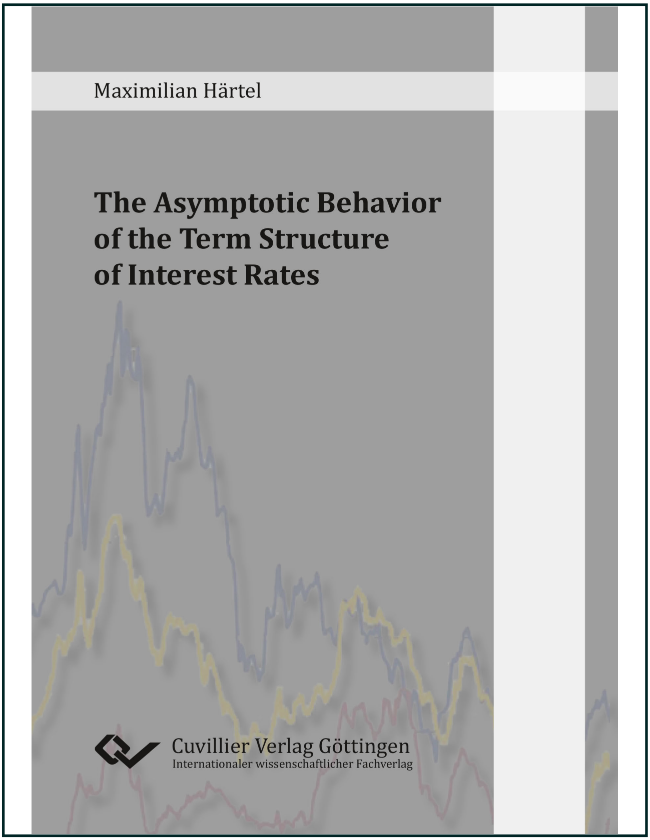
The Asymptotic Behavior of the Term Structure of Interest Rates



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1. Introduction

1.1. Motivation

A term structure can be defined as a function that puts a financial variable in relation to its maturity. Therefore, the term structure of interest rates relates interest rates or bond yields to different terms or maturities (cf. Chapter 1 of [83]). The term structure of interest rates is also called yield curve which is defined rigorously in our setting in Definition 2.2.3 and is assumed to be continuous. This curve is of fundamental importance in macroeconomics since it puts monetary policy in perspective to investment behavior resulting in economic growth and vice versa. It reflects the expectations of market participants about future changes in interest rates (cf. Section 1.2.3 of [58]). A practical problem is the determination of a mathematical expression for the current term structure because there is only a finite number of maturities of bonds that are traded at financial markets. This problem is solved by calibrating the term structure curve to current market data, i.e. by fitting a curve to a number of points, see, for example, Chapter 6 of [3] and Chapter 3 of [83]. To take into account the uncertainty in time evolution of the yield curve, stochastic interest rate models are needed that are coherent with market data. In the literature, there have been many different proposals for the modeling of interest rates, e.g. short rate models, interbank offered rate (IBOR) market models, swap market models, or the Heath-Jarrow-Morton (HJM) framework (cf. Chapter I and II of [33]). For the calculation of the yield at long maturities, the choice of the specific stochastic model that incorporates the expectations about the future behavior of the interest rates is crucial since there is minor market data for yields of longer maturities.[1] The asymptotic behavior of the yield curve as well as interest rates with a long term are very important topics for financial institutions that invest in products depending on a long time horizon, either via a maturity in the far-away future or due to perpetual characteristics. Therefore, the modeling of long-term interest rates is the subject of several publications in economical and mathematical research. Considering the various contributions to the topic, it has to be noted that no unique definition of long-term interest rates is provided. The denomination "long-term" can be understood in several different ways such that there exist different conventions on the concept of long-term interest rates in the literature. The *European Central Bank (ECB)* considers yields of government bonds with maturities of close to 10 years as long-term (cf. [78]), in [155] high-grade bonds with a time to maturity of more than 20 years are examined for the investigation of long-term interest rates, whereas in [161] the author considers the time span between 30 and 100 years for the analysis on the long end of yield curves. However, a natural mathematical approach to the study of long-term interest rates is to examine the different rates, meaning the continuously compounded spot rate, the simply compounded spot rate, and the swap rate, with their respective maturity going to infinity. This concept of long-term is used among others in

[1] For instance the longest term for United States (US) treasury yields provided by *Bloomberg* is 30 years (cf. [28]).

[23], [24], [25], [35], [40], [73], [160], and [161]. In this thesis we adopt this definition and present different convergence results for the long-term yield, the long-term simple rate, and the long-term swap rate dependent on the underlying interest rate model as well as results on the interdependencies between these rates that are independent of the model. All of these long-term interest rates are related to the long-term zero-coupon bond prices, which we define in our setting as the theoretical price of a perpetual zero-coupon bond, see Definition 3.1.1. The long-term yield ℓ is defined as the continuously compounded spot rate with maturity tending to infinity, see Definition 3.1.13, whereas the long-term simple rate L is the simply compounded spot rate with maturity tending to infinity, see Definition 3.1.17. This particular definition of the long-term yield corresponds to its definition in some textbooks such as in Section 2.3 of [38], in Subsection 6.3.2 of [40], and in [83]. The long-term simple rate was first defined this way in [35] to propose an alternative discounting rate for long-term financial products or projects with a very long time horizon. The long-term swap rate R, defined in equation (3.1.18), can be understood as the fixed fair rate of an overnight indexed swap (OIS) that has a payment stream with infinitely many exchanges. It should be fair in the sense that the price of the receiver and the payer of this OIS equals zero. This rate was defined for the first time in [24] and the use of an OIS as special case of an interest rate swap (IRS) stems from the fact that OIS rates are used as proxy for risk-free rates in interest rate modeling, due to the last financial crisis, as explained in Section 2.1. There is an ongoing debate about the starting point of this crisis, which is difficult to determine since the development of the crisis was a gradual process from the sub-prime crisis over to the credit crunch, then to the liquidity crisis of banks, and finally to the public debt crisis, especially in Europe (cf. [74] and [158]). Nevertheless, most of the literature considers the 9th of August 2007 as initial date, when *BNP Paribas*, one of the largest banks in the world, announced the closing of three hedge funds specialized in US mortgage debt. *BNP Paribas* was not able to value the holdings, in particular the collateralized debt obligations (CDOs). In consequence, the *ECB* allowed Euro area banks to draw as much liquidity as they needed for refinancing at the prevailing overnight rate on the same day (cf., for example, Chapter II of [13], [74], Section 3 of [75], [124], and [142]). However, there is a broad consensus about the climax of the financial crisis that is dated the 15th of September 2008, when the US investment bank *Lehman Brothers* filed for bankruptcy and a shock to the international financial market followed (cf. [45], [74], and [116]). Therefore, we call this crisis the 2008 financial crisis subsequently in this thesis.

In the course of this crisis term structure modeling in general changed significantly as explained in Section 2.1, and the evaluation of long-term financial products as well as interest rates became more important (cf. [19] and [76]). Besides the more mathematical approaches, a lot of studies were published addressing the topic of long-term interest rate modeling from a macroeconomic point of view. These approaches want to take into account the importance of monetary and fiscal policy regarding this subject, especially during the time of a financial crisis. Mankiw et al. examine in [129] the impact of monetary and fiscal policies on long-term interest rates and show that interest rates with a long time horizon do not react too sensitive to short-term rates. Several other economic factors can also be characterized as macroeconomic news, for example, data releases regarding the gross domestic product, new home sales, or initial claims as well as a substantial rise or decline in the unemployment rate or of the capacity utilization rate, see Table 1 of [102] for a comprehensive list. In general, the macroeconomic approaches to the topic of long-term interest rate modeling seek to identify precisely those factors influencing the long-term rates. Gürkaynak et al. provide in [102] evidence that most of the mentioned factors significantly affect both short-term and long-term rates, and in [107] the authors use an affine function dependent on macroeconomic variables to evaluate the continuously compounded spot rate. With the help of this model the influence of macroeconomic effects on the long-term yield can be measured. A three factor model is applied for modeling the yield curve in [123], i.e. the evolution of the interest rates is described by three latent factors that are employed in order to explain the empirical observation of falling long-term yields. The construction of a model that jointly characterizes the behavior of the yield curve and macroeconomic variables, is the subject of the publications [4] and [60]. In [4] a vector autoregression model is applied for the description of the relationship between interest rates and macroeconomics, whereas [60] uses a latent factor model with the inclusion of macroeconomic variables to model the yield curve. Instead of using other economic factors to explain the behavior of long-term interest rates, these long-term rates can also be understood as one of these factors that influence asset pricing. This approach is applied by Chen et al. in [41], where the long-term yield, in terms of long-term government bonds, is one of the several economic factors.

Regardless of considering a macroeconomic approach to the modeling of long-term interest rates or a more mathematical one, the obtained rate is essentially important for the pricing and hedging of long-term fixed income securities like perpetual bonds, life and accident insurances, pension funds, or IRSs with a long time to maturity. Besides these financial instruments there are situations in which the time horizon of cashflows extends beyond the limit of the observable term structure of interest rates: for example, the valuation of required financial resources for public and private retirement systems, the funding of long-term infrastructure projects, or the determination of compensatory adjustments in the course of an accident or a divorce. Therefore, as already mentioned, the knowledge about the asymptotic behavior of the term structure of interest rates is important from an economic as well as financial point of view. It allows to model a fair discounting rate for long-term products, but also to efficiently hedge these products. For instance, the consideration of the long-term swap rate is to some extent motivated by the observation that some financial products may involve the interchange of cashflows on a possibly unbounded time horizon since this rate could be useful in hedging the interest rate risk of these products. One of the products that are increasingly offered by banks since the start of the 2008 financial crisis is the contingent convertible (CoCo) bond. It is a debt instrument with an embedded option for the issuer, mostly banks, to convert debt into equity. This possibility is typically used by credit institutes that have to overcome a period of liquidity problems (cf. [1], [32], [64], [88], and [89]). In the course of the crisis Bernanke, the chairman of the *Federal Reserve* at that time, pointed out the importance of these products for financial institutions to maintain a certain level of capital, see [22]. Dudley, the president of the *Federal Reserve Bank of New York*, stated in [63] that, to strengthen the financial system, an increase in the use of CoCo bonds should be one of three main points realized in the aftermath of the 2008 financial crisis.[2] These products can be decomposed into a portfolio consisting of bonds and exotic options, see [32]. Furthermore, in [32] a valuation formula for the price of a CoCo bond with finite maturity

[2] The other two main points the financial industry should focus on, according to Dudley, are a more thorough and complete risk capture as well as rules that encourage institutes to save money in economic boom periods.

can be found, where in [1] the authors also consider the case of an infinite maturity. This is of practical importance since, besides CoCo bonds having a finite time to maturity, there also exist some with unlimited maturity. For example, in June 2014 *Barclays PLC* issued a perpetual CoCo bond, which pays fixed coupons of 7% with the investor having a first conversion possibility in 2019 and then every 5 years until eternity (cf. [147]). This perpetual product, where the coupons of the non-optional part are floating, could lead to investors seeking for a hedging instrument offering protection against the interest rate risk involved in the non-optional part of the contract. A fixed to floating interest rate swap with infinitely many exchanges could serve as a hedging product for the interest rate risk beared by CoCo bonds, with the fixed rate of such a swap being the long-term swap rate. Another motivation of analyzing the long-term swap rate lies in the context of multiple curve bootstrapping because, according to the post-crisis market practice, OIS contracts constitute the input quotes for bootstrapping procedures, which allow for the construction of a discounting curve, as explained in Section 2.1. In view of this, the long-term swap rate can be applied for inference of information on the long-end of the discounting curve. The main results of the investigation of the long-term swap rate R are an explicit model-free formula developed in equation (3.1.19) in the case of a convergent infinite bond sum S_∞, and Theorem 3.1.29, which tells us that this interest rate is either constant or non-monotonic over time. Further, we see in Corollary 3.1.25 that R is always finite if it exists. Hence, we propose the long-term swap rate as an alternative discounting tool for long-term investments, due to the facts that R is almost always finite, non-monotonic, can be explicitly characterized, and can be inferred by products existing on the markets.

In contrast to R, the long-term yield ℓ is monotonic in the sense that it is a non-decreasing process. This was first shown in [69] by Dybvig, Ingersoll, and Ross, and consequently is referred to as DIR theorem, see Theorem 3.1.16. It has been the topic of several publications, among others [96], [108], [121], and [132]. Besides Theorems 3.1.16 and 3.1.29 we are able to provide some more model-free results concerning the interrelations between different rates. All possible different relations between the three defined long-term interest rates are analyzed in Section 3.2. In this context we state the interesting fact that a strictly positive long-term yield entails a strictly positive long-term swap rate and an exploding long-term simple rate, see Corollary 3.2.2. Another intriguing relation is pointed out in Corollary 3.2.13 that tells us that if L is strictly positive it is not possible for ℓ and R to be strictly positive. Furthermore, we see in Corollary 3.2.19 that from a strictly positive long-term swap rate it follows that ℓ and L are non-negative processes in the rather realistic case of a finite long-term bond price. Apart from these general results on long-term interest rates, we also consider specific interest rate models to develop explicit formulas for ℓ, L, and R.

There are only a few studies analyzing long-term interest rates in predetermined term structure models and we try to provide a comprehensive overview of the different approaches in this thesis. Most of these approaches use a HJM framework like [23], [25], and [73]. In [73], El Karoui et al. examine the long-term yield in a Brownian HJM framework and conclude that in the case of a finite rate, it is independent from the underlying probability measure since the Brownian part vanishes (cf. equation (4.1.24)). This specific study of the long-term yield is presented along with results on L and R in this model in Section 4.1. In the following section, an extension of the Brownian HJM framework is considered by changing the stochastic driver from a Brownian motion to a Lévy process. This approach is based on [25] and its motivation stems from the idea to incorporate also jumps into the term structure model. Concerning the long-term interest rates in a Lévy HJM framework, the main results are findings regarding the asymptotic behavior of ℓ and Theorem 4.2.10. This theorem shows that the volatility part describing the long-term yield has to vanish, as in the Brownian HJM framework, if ℓ is not supposed to explode, except for the case of a Lévy process with only negative jumps and paths of finite variation serving as random driver. Then, in Section 4.3, another generalization of the HJM framework is used as a term structure model for the analysis of the long-term interest rates. The basis for this study is [23] and here, the stochastic drivers are affine processes on the state space of symmetric positive semidefinite matrices. This particular class of stochastic processes was chosen due to its appealing features such as the possibility to model correlated factors influencing the term structure's volatility or to describe positive spreads among different curves resulting from credit and liquidity risk. We want to take into account the increased study of this state space in financial research. A literature overview of this specific topic can be found at the beginning of Section 4.3. Again, we are able to provide an explicit formula for the long-term yield in this model, see equation (4.3.59). In this setting the long-term volatility part must also vanish if ℓ is supposed to exist finitely as proven in Theorem 4.3.16. The asymptotic behavior of the other long-term interest rates L and R is concluded from ℓ. Following this approach, we consider some interest rate models that are not assigned to the HJM framework in the next two sections. These are the Flesaker-Hughston model and the linear-rational methodology. The consideration of long-term interest rates for both of these models is part of the article [24] by Biagini et al. In Section 4.4, the Flesaker-Hughston term structure model, which is named after the authors of [91], is described and applied for the valuation of long-term interest rates. This model was developed in 1996 and has been the topic of several publications (cf. e.g. [97], [133], [139], [151] and [153]), due to some favorable characteristics such as relatively simple resulting models for bond prices, short and forward rates. Other advantages are the specification of only non-negative interest rates and the high degree of tractability. The different long-term interest rates are computed in two specific examples of this approach, where the functions specifying the form of the zero-coupon bond prices are given. Then, in Section 4.5 we present the linear-rational term structure methodology, which was recently introduced in [84] by Filipović and Trolle, as basis for the investigation of long-term interest rates. This class of term structure model was chosen for our considerations of long-term interest rates since it presents various advantages: it is highly tractable, non-negative interest rates are guaranteed, unspanned factors affecting volatility and risk premia are accommodated, analytical solutions to swaptions are admitted, and it offers a very good fit to IRSs and swaptions data. The main result here is a closed-form formula for the long term swap rate, see equation (4.5.9), and the fact that ℓ is a constant process.

Altogether, this thesis presents a complete discussion of the asymptotic behavior of the term structure of interest rates. All different long-term interest rates are defined and characterized after the introduction of the necessary interest rates and fixed income products in a modern modeling framework. Then, the interrelations of the long-term interest rates and their other model-free properties are explained to finally compute the rates explicitly and study their asymptotic behavior in specific and appropriate term structure models.

The thesis is structured as follows. Chapter 2 introduces the setting for interest rates and fixed income products necessary for all further investigations. For this matter, we first distinct in Section 2.1 the interest rate modeling in the post-crisis era from former one-curve frameworks. Then, we describe in Section 2.2 all interest rates needed in the course of the thesis by considering the discount curve of a multi-curve framework since we are mainly interested in long-term interest rates as discounting tool. Section 2.3 gives insight about the pricing of collateralized contracts such as IRSs, whose evaluation formula is derived in detail in Section 2.4. In particular, the computation of an OIS rate is shown, which is needed for the definition of R. Chapter 3 deals with long-term interest rates in the sense that they are defined and analyzed with regard to universally valid properties and their relations towards each other. In Section 3.1 the reader finds the definitions and universal characteristics of the long-term bond price P, of the infinite sum of bond prices S_∞, as well as of the long-term interest rates ℓ, L, and R, whereas the interrelations are described in Section 3.2. Finally, in Chapter 4 we analyze the long-term interest rates in some selected term structure models. The structuring of the different sections within this chapter has already been explained in detail above.

1.2. Contributing Manuscripts

This thesis is based on the following manuscripts which were developed by the thesis' author M. Härtel in cooperation with co-authors:

1. F. Biagini and M. Härtel [25]: *Behavior of Long-Term Yields in a Lévy Term Structure. International Journal of Theoretical and Applied Finance, 17(3): 1-24, 2014.*

   The results of this publication on the behavior of long-term yields in a term structure model using Lévy processes as stochastic driver were devised by M. Härtel together with Prof. F. Biagini. The work was developed at the LMU Munich. The suggestion of investigating the asymptotic behavior of the yield curve in a Lévy HJM framework was made by Prof. F. Biagini in order to to generalize the approach of N. El Karoui, A. Frachot, and H. Geman in [73] who considered a Brownian HJM framework for their analysis. A significant part of the computations contained in the proofs was taken care of by M. Härtel.

2. F. Biagini, A. Gnoatto, and M. Härtel [23]: *Affine HJM Framework on S_d^+ and Long-Term Yield. LMU Mathematics Institute, Preprint, 2013.*
   Available at: http://www.fm.mathematik.uni-muenchen.de/download/publications.

   This article is a joint work of Prof. F. Biagini, Dr. A. Gnoatto, and M. Härtel. It was developed at the LMU Munich. In joint discussions, we developed the idea of considering affine processes on the state space of symmetric positive semidefinite matrices S_d^+ for an analysis of the asymptotic behavior of long-term yields. These kind of processes have appealing features for term structure modeling and are used frequently in recent publications of financial research. Sections 2 and 3, where necessary results on affine processes on S_d^+ are gathered and the HJM framework for this kind of driving process is described, were developed by M. Härtel with support by Dr. A. Gnoatto. The investigation of the long-term yield in this particular framework which is the content of Section 4 was developed in close cooperation by Prof. F. Biagini, Dr. A. Gnoatto, and M. Härtel. The examples presented in Section 5 were chosen and computed independently by M. Härtel.

3. F. Biagini, A. Gnoatto, and M. Härtel [24]: *The Long-Term Swap Rate and a General Analysis of Long-Term Interest Rates. LMU Mathematics Institute, Preprint, 2015. Available at: http://www.fm.mathematik.uni-muenchen.de/download/publications.*

   This paper defining for the first time in literature the long-term swap rate and analyzing the interrelation of long-term yield, long-term simple rate, and long-term swap rate emerged by a collaboration of Prof. F. Biagini, Dr. A. Gnoatto, and M. Härtel. The article was developed at the LMU Munich. In joint discussions we developed the idea of introducing the long-term swap rate as a new kind of long-term interest rate. Dr. A. Gnoatto and M. Härtel have embedded this approach in the context of the post-crisis interest market by using the fact that OIS rates are mainly used as discounting rates in multi-curve frameworks. Sections 2 and 3, where prerequisites for further examinations are stated, were developed by M. Härtel. Sections 4 and 5 that contain the main results of the paper by defining and characterizing the long-term swap rate as well as investigating all relations between the long-term interest rates were developed in a joint work by Prof. F. Biagini, Dr. A. Gnoatto, and M. Härtel. The analysis of the long-term interest rates in two specific term structure models in Section 6 was performed by M. Härtel with help by Dr. A. Gnoatto.

The following list indicates in which way the three publications contribute to each part of the thesis. The formulation of the statements of the corollaries, definitions, lemmas, propositions, and theorems is similar or the same as in the three manuscripts. However, the author, who has been involved in the development of all the results contained in the three articles, provides in the present thesis a more detailed version for most of the proofs.

1. Chapter 1 was developed independently by M. Härtel.

2. Chapter 2 was developed independently by M. Härtel.

3. Chapter 3 is mainly based on F. Biagini, A. Gnoatto, and M. Härtel [24]. Section 3.1 consists of Sections 4 and 5 of F. Biagini, A. Gnoatto, and M. Härtel [24] and some work that was done independently by M. Härtel. Section 3.2 is based on Section 5 of F. Biagini, A. Gnoatto, and M. Härtel [24].

4. Chapter 4 is based on all three manuscripts [23], [24], and [25]. Section 4.1 was developed independently by M. Härtel and provides the basis for the following sections by illustrating in details some results of N. El Karoui, A. Frachot, and H. Geman [73]. Section 4.2 is based on Sections 2 and 3 of F. Biagini and M. Härtel [25]. Section 4.3 is based on Sections 2 - 4 of F. Biagini, A. Gnoatto, and M. Härtel [23]. Section 4.4 is based on Subsection 6.1 of F. Biagini, A. Gnoatto, and M. Härtel [24]. Section 4.5 is based on Subsection 6.2 of F. Biagini, A. Gnoatto, and M. Härtel [24].

2. Fixed Income Basis

In this chapter we collect some basic results and notations of instruments used on fixed income markets. The purpose is to develop a common language for the remainder of the thesis. For this, we have to introduce notations to characterize prices and yields of basic fixed income market securities as zero-coupon bonds, the money-market account and different interest rates. The interrelation between these securities is addressed as well as their respective significance in fixed income markets. When speaking about these different instruments it is important to be clear about which curve is used since the main difference between the theory of interest rate modeling before and after the 2008 financial crisis is, that in the post-crisis framework there is not only one curve used for discounting and computing forward rates, which is assumed to be risk-free, but one discounting curve representing the risk-free curve and multiple curves for modeling the forward rates dependent on the respective instrument. Therefore, we first distinguish the classical single-curve approach from the modern multi-curve framework of interest rate modeling in Section 2.1 and provide the reader insight into the effects of the financial crisis on yield curve modeling. Then, in Section 2.2 the needed interest rates as well as zero-coupon bonds are explained in a multi-curve framework. In the course of our investigations on long-term interest rates, we will consider long-term swap rates that depend upon a special class of IRSs, namely OISs. For this reason, we examine IRSs with regard to valuation of their present values and corresponding forward swap rates in Section 2.4. In the preceding section the collateral is defined and a formula for the present value of collateralized financial instruments is presented since collateralization is crucial for the valuation of derivatives, especially IRSs. Throughout the whole thesis we consider a probability space $(\Omega, \mathscr{F}, \mathbb{P})$ endowed with the filtration $\mathbb{F} := (\mathscr{F}_t)_{t\geq 0}$ satisfying the usual hypothesis of right-continuity and completeness, where $\mathscr{F}_\infty \subseteq \mathscr{F}$ and $\mathscr{F}_0 = \{\emptyset, \Omega\}$. The probability measure $\mathbb{P}$ denotes the real-world measure and the equivalent probability measure $\mathbb{Q}$ is the risk-neutral one.

Note, that for the valuation of fixed income instruments and derivatives the time between the observation date and a future cashflow plays an important role. This time span is always measured in years but this year fraction depends on the use of a specific calendar, the business day convention, and the day-count convention. The choice of calendar determines the holidays in the respective payment schedule, whereas the business day convention explains how to adjust for dates in the payment schedule if they fall on a day that is not a business day. The day-count convention describes the method of calculating an accrual factor that relates to a given period. That means, when $\tau(s,t)$ measures the time between two dates $0 \leq s \leq t$ it must contain the information about these time counting conventions. A full discussion of the different business day conventions and day-count conventions can be found in Section 4.3 of [42] or in Chapter 2 of [81]. In Appendix B of [106] the market conventions for IBOR indexes, overnight indexes, and different fixed income products and derivatives are listed. Another very important factor that influences the valuation of financial instruments is the use of an exchange rate if the considered instrument is denoted in another currency than its numéraire. We will only consider a unique currency in the thesis but all following results could be converted to foreign currencies by the formulas presented in Section 2.2.1 of [92].

For the description of the single-curve framework we follow the textbooks [3], [33], and [83], whereas the main references regarding the multiple curve setup are [26], [50], [92], [106], and [135]. The information about collateralization and central clearing is taken from [6], [34], [87], [104], and [146]. We primarily used results of [2] and [87] for the discussion of IRSs.

2.1. Post-Crisis Interest Rate Market

Interbank risk can be defined as lending risk in the interbank money market according to Definition A.3. The importance of interbank risk grew in the course of the recent financial crisis and influenced the modeling of interest rates significantly. It is measured as the spread between an IBOR and the rate of a maturity-matched OIS. An IBOR is the interest rate at which banks lend to and borrow from one another in the interbank market. In the USD-denominated fixed income market the main reference rate is the USD London interbank offered rate (LIBOR), whereas in the EUR-denominated fixed income market this rate is the European interbank offered rate (EURIBOR). Both of these rates are derived as a trimmed average of specific bank panels that are periodically reviewed and revised. There are also IBORs for all kinds of local rates, like for example STIBOR for the SEK rate fixed in Stockholm (cf. Section 1.1.1 of [122]). IBORs are quoted for a range of maturities with the most important being overnight, three and six months, denoted by 1D, 3M, and 6M (cf. Section 1.1.1 of [122]).[1] There are as well fixed income products or derivatives such as OISs that are tied to overnight rates. In the USD market, the main reference rate is the effective Federal funds (FF) rate and in the EUR market the benchmark is the Euro overnight index average (EONIA) rate (cf. Section 2.1 of [87]). An OIS is an IRS where a fixed rate for a period is exchanged for the geometric average of overnight rates during this period.[2] The mentioned FF rate and EONIA rate are the overnight rates used in the OIS geometric average calculations. A party can swap its overnight borrowing or lending for borrowing or lending at a fixed rate, whereby this fixed rate is referred to as OIS rate. The calculation of this rate is explained in detail in Section 2.4. To get insight how the increased interbank risk changed the way of term structure modeling, we first consider the pre-crisis modeling approach.

Before the 2008 financial crisis the term structure of interest rates was modeled via a single-curve approach. The approach's name stems from the fact that one risk-free curve was used for modeling the discounting and forward rates. The concept of risk-free refers to the absence of elements of credit and liquidity risk, not to the absence of interest rate risk.[3] After constructing this risk-free yield curve it reflects at the present the costs of future cash flows as well as the level of the forward rates (cf. [33], [83], and [109]). This approach was justified by negligible coun-

[1] For more information on LIBOR and EURIBOR, especially on the composition of the bank panels and the different maturities, refer to [79] and [113].

[2] As noted in Section 9.2 of [110] the term "geometric average of overnight rates" must be interpreted as "geometric average of one plus the overnight rates minus one".

[3] The terms credit, interest rate, and liquidity risk are explained in Definitions A.1, A.4, and A.5.

terparty and liquidity risk (cf. Section 1 of [141]).[4] The crisis caused several inevitably consequences for all financial market participants, as among others, companies were faced with credit and liquidity problems more than ever. Hence, credit and liquidity risks had to be accounted for when pricing financial products what resulted in increased spreads between different tenors and currencies as well as in increased credit spreads. These spreads were typically smaller than the bid-ask spread and therefore negligible before the start of the crisis (cf. Section 2.3 of [26]). The definitions and corresponding interpretations of these spreads can be found in Appendix B. Figures 2.1 and 2.2 capture the growth of different credit spreads, in fact the IBOR-OIS spreads and TED spreads for the USD and EUR markets. In Figure 2.1 the LIBOR-OIS and EURIBOR-OIS spreads for 3M rates are displayed in basis points (bps) over a nine-year time period beginning in the first quarter of 2006 and ending in the first quarter of 2015.[5] It can be seen that before the start of the financial crisis the spreads were very low indicating almost no default risk in the interbank market. Between the end of the first quarter of 2006 and the beginning of August 2007 the 3M LIBOR-OIS spread was never larger than 15 bps, and the 3M EURIBOR-OIS spread never exceeded 10 bps, only to jump to 39,95 bps and 17,7 bps, respectively, on the 9th of August 2007 what is often considered as the crisis' start, as explained in Section 1.1. Then, after a continuous increase, the IBOR-OIS spreads peaked in the aftermath of *Lehman Brothers*' bankruptcy with 364,43 bps for the LIBOR-OIS spread and 195,50 bps for the EURIBOR-OIS spread. Following this peak the IBOR-OIS spreads settled at a much lower, but nevertheless non-negligible, level with an interim high in December 2011 during the climax of the European sovereign crisis (cf. Section 1 of [145]).

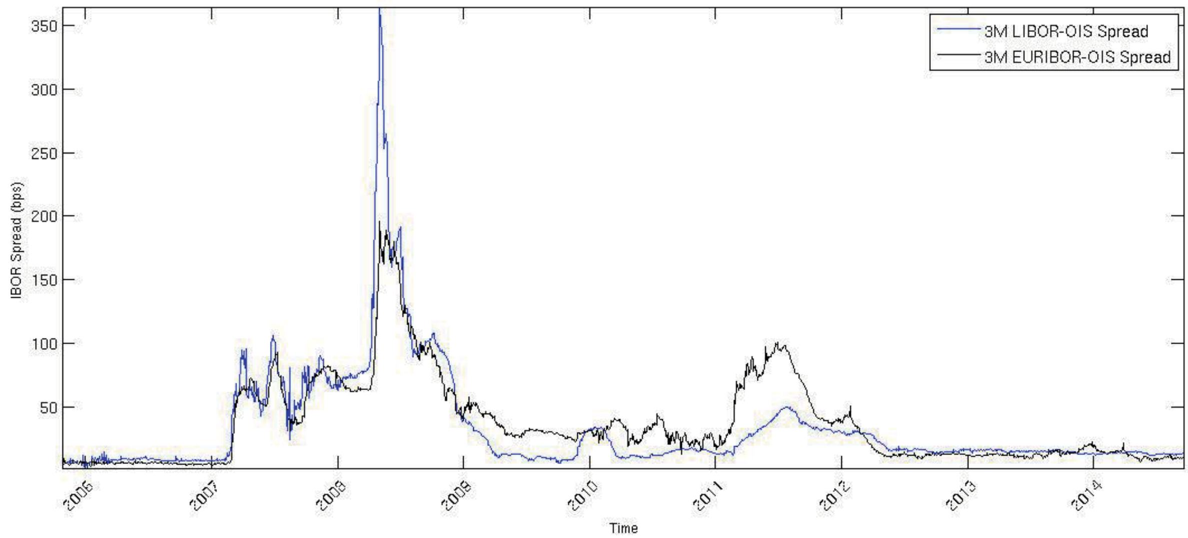

Figure 2.1.: 3M LIBOR and 3M EURIBOR spreads. Own presentation, data retrieved from *Bloomberg*.[6]

[4] A description of counterparty risk can be found in Definition A.2.
[5] One bp equates to one hundredth of a percentage point, i.e. $1\,\text{bp} = 0{,}01\%$ (cf. Section 13.12.1 of [33]).
[6] The author is grateful to *IDS GmbH - Analysis and Reporting Services* for providing the *Bloomberg* data.